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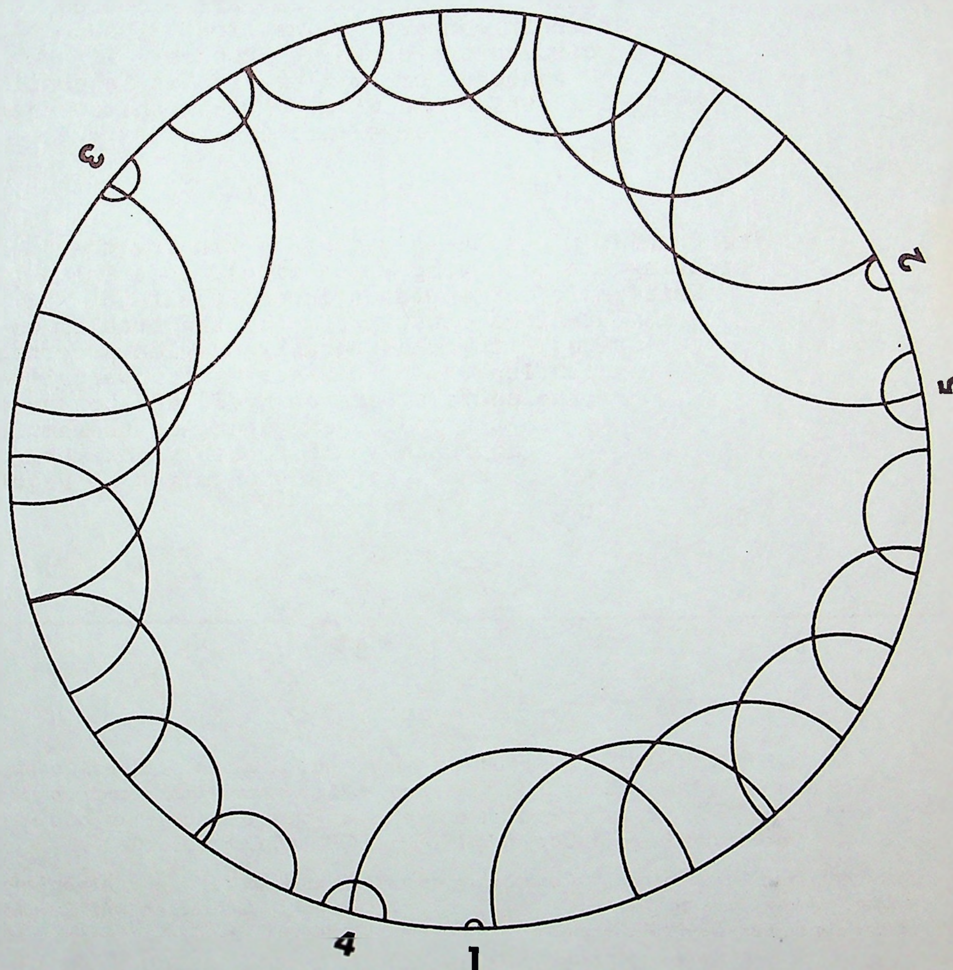

The Obfuscating Circles

THE OBFUSCATING CIRCLES

The large circle on the cover has a diameter of 6 units. The smaller circles have diameters of .1, .2, .3, .4,... units, and are centered at points that are 6 units apart on the circumference of the large circle (that is, at intervals of every two radians of central angle). The first 5 of the circles are numbered at their centers. The first 25 such circles are shown. Problem: if the sequence of smaller circles is continued, which one will complete the coverage of the large circle?

The Problem might be solved by continuing the drawing and observing which circle does the job. This empirical approach lends itself to computerized plotting. For the problem as stated, it would probably provide a correct solution. If, however, the interval between the centers were changed, and/or the increment in the radii were changed, the empirical approach would probably not work, and an analytic solution might be required. □

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N-SERIES 25

Log 25 1.397940008672037609572522210551013946463620237075782917

Ln 25 3.218875824868200749201518666452375279051202708537035444

$\sqrt[3]{25}$ 2.924017738212866065506787360137922778530498635101030041

$\sqrt[5]{25}$ 1.903653938715878489896147288119097778655062586108560553

$\sqrt[7]{25}$ 1.583819608766579044552643382752747852313248105374232359

$\sqrt[10]{25}$ 1.379729661461214832390063464216017692855649877977606122

$\sqrt[100]{25}$ 1.032712419896443050915132358204671897789875147184427347

e^{25} 72004899337.385872524161351466126157915223533813395278736221386447232059310778256974500032565425809[3]

π^{25} 2683779414317.764549009928124395386777953247850874391888511221809941911179548161762161966865280688[6]

$\tan^{-1} 25$ 1.530817639671606577817743842010376038501565652495829422

25^{100} 6223015277861141707144064053780124240590252168721167133101116614789698834035383441183944823125713616956966589555122482124716043472290039062[5]

N-SERIES 9

The usual values for entry 9 in the N-series were omitted in favor of other interesting numbers. To make the series complete, the regular values are given here:

$\sqrt[3]{9}$ 2.080083823051904114530056824357885386337805340373262109

$\sqrt[5]{9}$ 1.551845573915359674273345135516699323262346293809667838

$\sqrt[7]{9}$ 1.368738106642201674842367788664029653049786979819082599

$\sqrt[10]{9}$ 1.245730939615517325966680336640305080939309993068779811

$\sqrt[100]{9}$ 1.022215413278477020829316576243419810698983623819729905

e^9 8103.083927575384007709996689432759965011476087831613462500159052178272515690624828686451092403461447077482

π^9 29809.099333446211666509402401239655363868057774288653316671156027854610833913152209524765910571049166090[2]

$\tan^{-1} 9$ 1.460139105621000972672181819429689336123298604684488878

9^{100} 265613988875874769338781322035779626829233452653394495974574961739092490901302182994384699044001

3X+1 Once Again

The 3X+1 problem has been discussed in issues 1, 4, and 13. The problem is this:

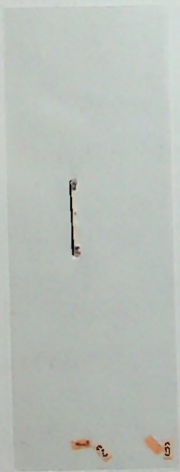
> Take a positive integer, N. Let X equal N. If X is odd, replace X by 3X+1. If X is even, replace X by X/2. Continue this process until X equals one. Let A equal the number of terms so generated, including the original value.

The accompanying table shows the tabulations of the A values for all N from 1 to 100000 (top) and for all N from 100001 to 200000 (bottom), from calculations made by Lynn Wyatt. 163 small values are missing from the top table; the largest omitted value is 1 for A = 351. 166 small values are missing from the bottom table; the largest omitted value is 1 for A = 443.

In the first writeup on the 3X+1 problem (in PC1), the statement was made "...not every A value can be produced, and empirical studies indicate that relatively few A values exist." This new evidence seems to vitiate that statement, but the distribution of the A values is nevertheless erratic.

A table (PC1-3) showed the first appearance of successively larger values of A, with their corresponding N, obtained during a run in which only odd values of N were being examined. A new entry for that table has been found:

A = 706 N = 31466383



The Chips Gap

In 1959, a delegation of U.S. computer experts visited the Soviet Union and some of its computing facilities. Paul Armer brought back a pocketful of chips from a 513 reproducer, a few of which are attached to the initial print run of this issue.

The card-punching machine was one of many loaned to Russia as part of World War II lend-lease. An IBM customer engineer would notice that its dies have not been kept sharp; the edges of the chips are very fuzzy. In addition, the card stock is repulped and of lower quality than we are accustomed to.

	0	1	2	3	4	5	6	7	8	9
000	0	0	1	1	2	1	2	2	4	4
010	6	6	8	10	14	18	24	29	35	43
020	57	63	81	80	105	138	124	171	186	195
030	268	201	289	416	290	428	261	408	618	371
040	571	629	483	782	400	652	1050	510	863	384
050	672	1148	496	879	935	660	1158	458	847	1337
060	588	1101	391	756	1424	503	975	703	637	1250
070	408	831	1467	536	1073	343	720	1417	439	908
080	252	559	1181	341	729	1130	467	1002	287	622
090	1234	385	820	227	499	1060	312	676	878	421
100	860	255	555	1091	351	719	205	429	938	283
110	614	175	380	807	234	492	870	311	647	207
120	430	919	296	603	210	417	826	292	579	609
130	405	778	295	559	1052	383	730	263	502	959
140	337	658	297	468	910	300	593	1148	389	780
150	245	501	995	313	645	202	417	853	252	536
160	782	337	711	193	431	913	250	563	141	329
170	753	199	434	149	254	576	150	338	745	195
180	452	109	259	609	149	351	84	198	481	114
190	278	573	150	360	73	195	472	93	246	47
200	128	340	55	163	82	74	226	37	121	319
210	58	165	18	63	205	27	92	9	36	129
220	19	62	195	27	97	9	38	126	19	59
230	6	24	90	11	40	43	22	68	12	35
240	96	15	46	11	32	77	15	42	6	19
250	51	9	29	72	13	43	6	20	52	7
260	28	4	11	36	7	16	32	9	26	5
270	11	30	6	20	2	8	29	3	11	2

010	0	0	0	0	0	0	0	0	1	1
020	0	8	0	23	27	0	57	27	48	118
030	0	171	220	0	301	1	359	486	0	553
040	366	238	825	0	823	1214	0	1161	0	1044
050	1633	0	1426	933	581	1906	0	1540	2106	180
060	2043	0	1598	2737	0	2085	736	1140	2728	0
070	1985	2993	174	2544	0	1747	3278	0	2312	0
080	1576	3040	0	2057	2629	423	2675	0	1756	3234
090	67	2279	0	1457	2907	0	1892	2126	532	2491
100	0	1556	3010	66	2064	0	1307	2733	0	1708
110	0	1118	2315	0	1473	2243	201	1931	0	1209
120	2584	0	1624	0	1017	2155	0	1371	1393	528
130	1904	0	1252	2464	16	1718	0	1161	2331	0
140	1557	242	1014	2134	0	1471	2775	25	1939	0
150	1309	2595	0	1705	0	1109	2249	0	1495	1897
160	337	1967	0	1269	2530	11	1697	0	1084	2253
170	0	1428	187	808	1827	0	1112	2357	8	1488
180	0	909	1992	0	1207	0	718	1593	0	961
190	1873	47	1248	0	750	1639	5	1010	0	581
200	1347	0	778	216	360	991	0	552	1306	3
210	739	0	405	964	0	530	0	267	710	0
220	364	906	2	507	0	271	694	0	349	0
230	174	473	0	251	185	82	341	0	178	464
240	0	230	0	118	328	0	189	0	94	253
250	0	133	354	0	179	0	103	263	0	128
260	0	67	180	0	93	170	13	130	0	68
270	193	0	95	0	56	157	0	79	2	32
280	100	0	46	138	0	63	0	31	91	0
290	46	0	20	58	0	24	60	2	40	0
300	22	67	0	30	0	18	53	0	27	1
310	13	40	0	19	53	0	24	0	11	32
320	0	14	0	7	22	0	12	32	0	20

Isolated terms of the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,...)

50 12586269025
51 20365011074

100 354224848179261915075
101 573147844013817084101

150 9969216677189303386214405760200
151 16130531424904581415797907386349

200 280571172992510140037611932413038677189525
201 453973694165307953197296969697410619233826

250 7896325826131730509282738943634332893686268675876375
251 12776523572924732586037033894655031898659556447352249

300 22223224462942044552973989346190996720666693909649976499
0979600
301 35957932520658356096176566517218909905236721430926723225
5589801

350 62544494288205516415497721901701841906081775146743317264
39961915653414425
351 10119911756749018713965376799211044556615579094364594923
736162239653346274

400 17602368064501396646822694539241125077038438330449219188
6725992896575345044216019675
401 28481229810848961175798893768146099561538008878230489098
6477195645969271404032323901

450 49539670118750664731625249252316040477277918713460610011
50551747313593851366517214899257280600
451 80156870043596115037168387733153212558390773036997994982
82226546635165089515533148342062946549

500 13942322456169788013972438287040728395007025658769730726
4108962948325571622863290691557658876222521294125
501 22559151616193633087251269503607207204601132491375819058
8638866418474627738686883405015987052796968498626

Fibonacci

The ratio of the 501st term to the 500th term is

1.61803398876955

which agrees to 11 significant digits with the golden mean, $(1+\sqrt{5})/2$.

The Generalized Birthdate Problem

For how many people, taken at random, will it be an even money bet that some two of them have the same birthdate?

This is a well-known problem, for which the answer is 23. The calculation is done as follows:

$$\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\frac{362}{365}\;\cdots\;\frac{343}{365}$$

.99726

.99180

.98364

.49270

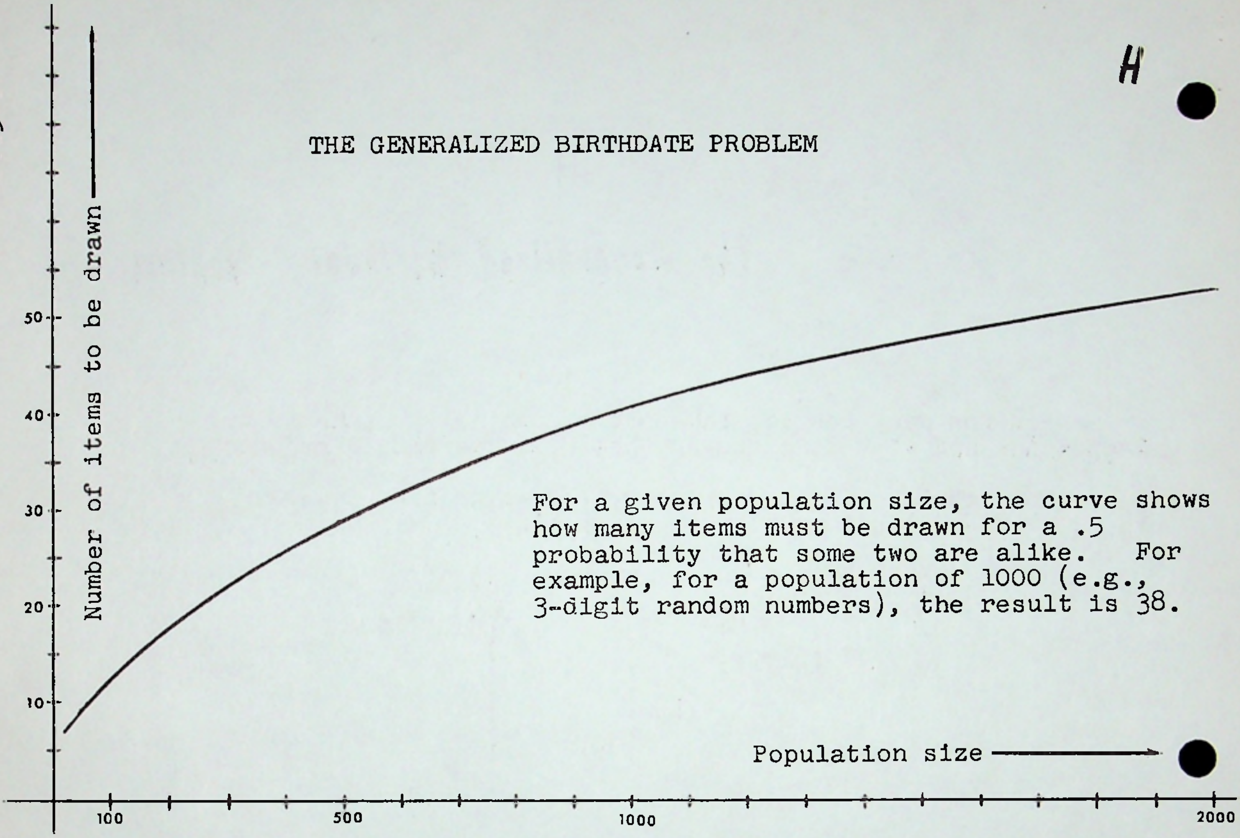
by the following reasoning. We calculate the probability of having, out of K people, all <u>different</u> birthdates (and the probability we seek is then the complement of this result). We begin with K = 1, for which the probability of having K dates all different is 1.000, expressed as 365/365, since we are working with a population of 365 things. For K = 2, there are 364 possibilities remaining in order to maintain all different birthdates, so the net probability for K = 2 is the probability for K = 1 multiplied by 364/365. The calculation proceeds as shown, until the net probability falls <u>below</u> .5 (at K = 23), at which point the probability we are seeking must be <u>over</u> .5.

The problem as stated has two parameters; namely, the population size (365 for the birthdate problem) and the probability level (.50). If we keep the probability level constant at .50 and vary the population size, we get the results shown in Figure H. Thus, for a population of 100, the number of items drawn is 13. This is the license plate problem. Of 13 cars taken at random, the probability is .5 that some two of them will have their last two digits the same.

PC25-8

H

THE GENERALIZED BIRTHDATE PROBLEM



Or, for a population of 1000, we have this: of 38 cars taken at random, the probability is .5 that some two of them will agree on the last three digits of their license numbers.

As a rough rule of thumb, the value of K for an even money bet is 1.2 times the square root of N, where N is the population size. For a population of 3000, this formula gives K = 66 (the true value being 65).

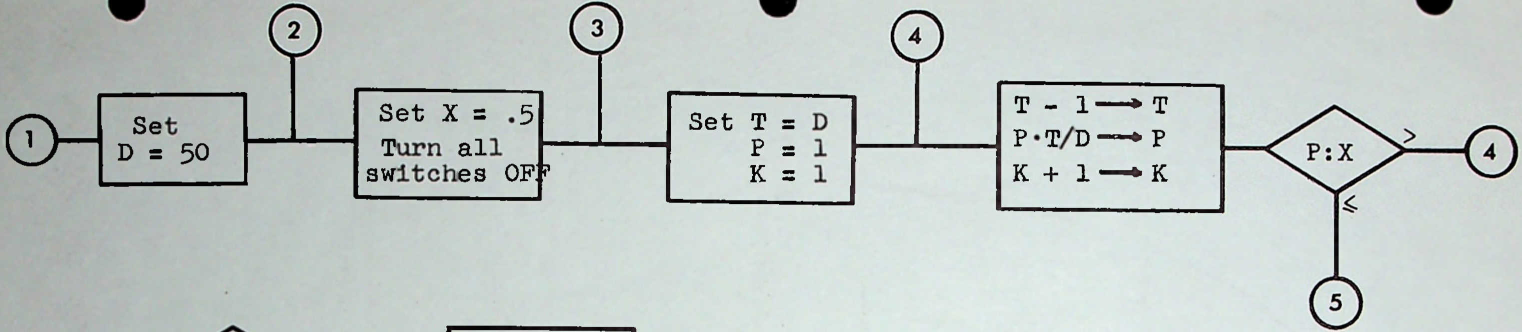
For N = 30, the formula gives K = 7. This says that seven people have a .5 probability of having the second hand of their wrist watches in agreement within one second. If they can read their watches to within half a second, then N = 60, and K = 10.

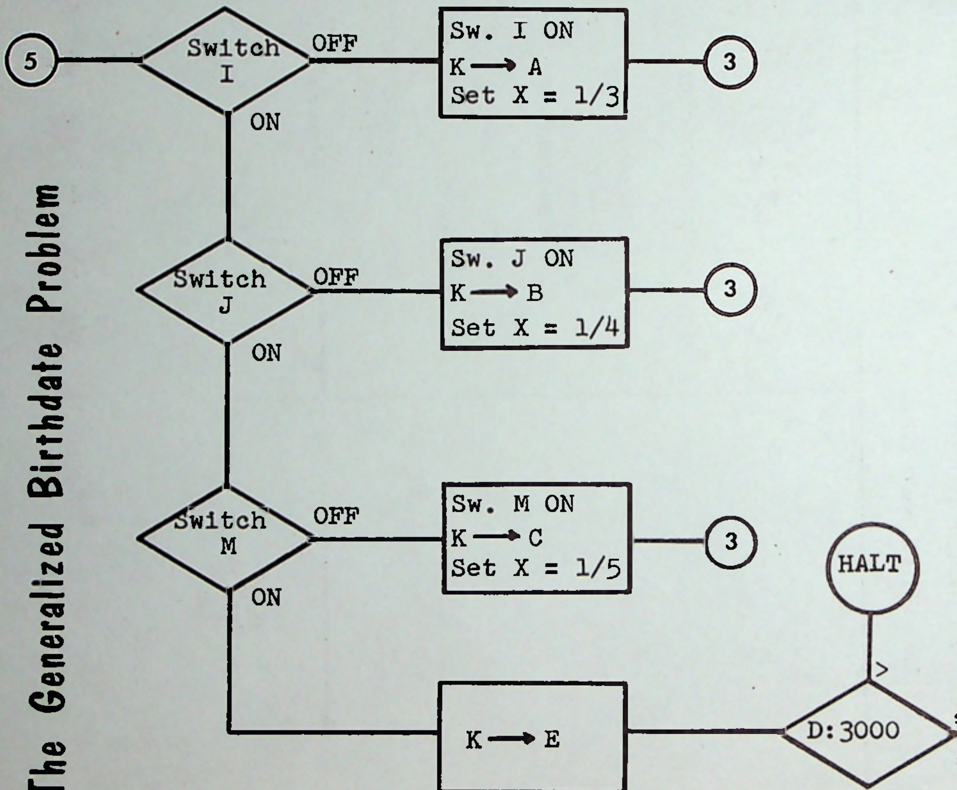
The other parameter of the generalized problem, the probability level, can be varied also. The accompanying flowchart shows a scheme of calculation for probability levels of 1/2, 2/3, 3/4, and 4/5, for population sizes from 50 to 3000 by 50's.

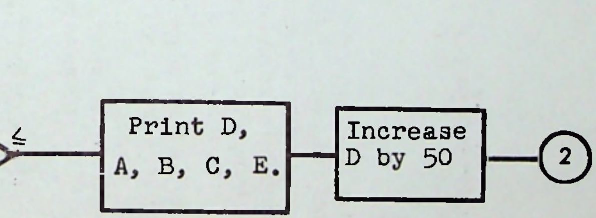
The Problem is this: What are corresponding rules of thumb for probability levels other than .5 for the generalized problem?

PROBLEM 88

The Generalized Birthdate Problem







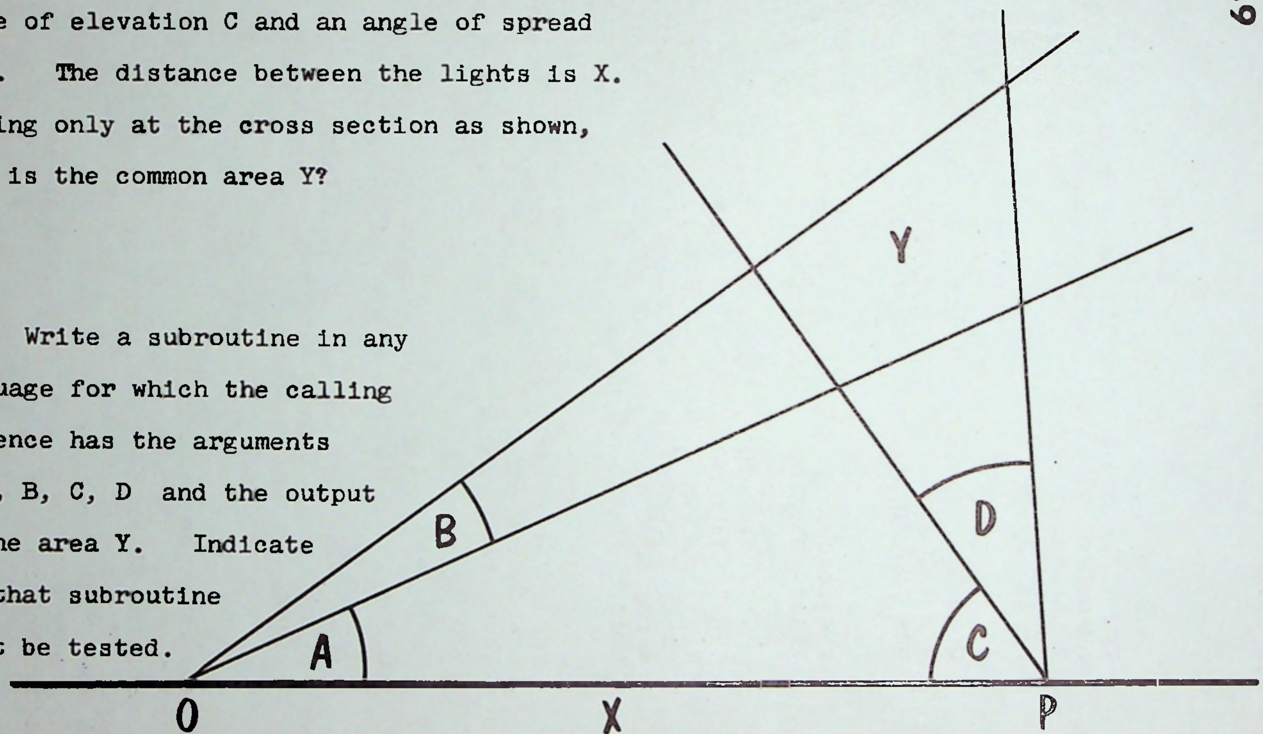
D is the population size; it is also the denominator in the calculations.
T is the numerator of the fractions used in the calculations.
P is the probability being calculated.
X is the probability level to be reached.
K is a counter of the number of draws from the population D.
A, B, C, and E are the results for probability levels of .50, .667, .75, and .80 respectively. In the calculations, X takes on values of .50, .333, .25, and .20, since we proceed by calculating the complement probability.

PROBLEM 89

Searchlight

Two searchlights are located at O and P. The searchlight at O makes an angle A with the horizontal and sends out light in a sector of angle B. The searchlight at P has an angle of elevation C and an angle of spread of D. The distance between the lights is X. Looking only at the cross section as shown, what is the common area Y?

Write a subroutine in any language for which the calling sequence has the arguments X, A, B, C, D and the output is the area Y. Indicate how that subroutine might be tested.



More on Penny Flipping

In PC23-10, the first three Penny Flipping problems were given, with some known results. Let us state all the problems.

In each case, a group of coins is given, all heads up. The coins are flipped in small sub-sets, and the number of flips that must be made until all coins return to heads is counted. The number of flips is a function of the number of coins.

I. (The original problem, from the book SIMULA BEGIN.) Given a stack of N pennies. Flip the top 1, the top 2, the top 3,..., the entire stack, then the top 1, the top 2, the top 3,..., the entire stack, and so on.

II. For a stack of N pennies, flip the top 1, the bottom 2, the top 3, the bottom 4,..., the entire stack, the top 1, the bottom 2, and so on.

III. The pennies are arranged in a triangular array (thus there are N rows, but the number of pennies is one of 1, 3, 6, 10, 15, 21, etc.). Flip one row, then 2 rows, 3 rows,..., the entire array. Then rotate the array 120° clockwise and start over, flipping the top row, then 2 rows, and so on.

IV. For a stack of N pennies, flip the top 1, then the entire stack, the top 2, the entire stack, the top 3, the entire stack,..., until the top K is the entire stack; then start over with the top 1, the stack, the top 2, and so on.

V. For a triangular array, flip one row, rotate the array, then 2 rows, rotate, 3 rows, rotate, and so on.

VI. For a stack of N, flip the top 1, the entire stack, the bottom 2, the entire stack, the top 3, the stack, the bottom 4, and so on.

Results were given for problems I and II for the first 32 values of N. Problem III is dull, since the functional value is 3N or 3N-1 for even and odd N respectively. Problem V is also dull; the functional value is 2N when N is congruent to 0 or 3 mod 6, is 3N when N is congruent to 2 or 4 mod 6, and is 3N-1 when N is congruent to 1 or 5 mod 6.

Problems II and IV seem to be the interesting ones. The accompanying table shows results for these two problems; 31 new values for PF-II, and 34 values for PF-IV. The results shown here were calculated by Thomas Sardi.

Both of these functions are wild indeed, and the preliminary results for PF-IV indicate that it is the wilder of the two. The case for N equals 58 in PF-II is still unknown.

Clearly, more results will be needed before anyone could establish a relationship between N and f for either case. Both of these problems are readily coded in any language for any computer.

II

N	f
33	20790
34	28560
35	16170
36	16839
37	16872
38	83980
39	13104
40	15960
41	30668
42	5880
43	13330
44	230384
45	62100
46	2484
47	1410
48	264960
49	16316
50	12900
51	27744
52	2600
53	2226
54	1156680
55	75889
56	11760
57	13680
58	
59	14868
60	140400
61	14640
62	135408
63	232848
64	25535

IV

N	f
4	17
5	40
6	27
7	18
8	97
9	43
10	531
11	200
12	219
13	312
14	2184
15	437
16	2700
17	501
18	1088
19	15120
20	608
21	1267
22	923
23	1848
24	828
25	2112
26	1500
27	1144
28	1443
29	2688
30	51040
31	12000
32	5208
33	14336
34	55440
35	1904
36	32743
37	8640

Problem Solution

Problem 54-K (PC16-13) called for extending the sequence

5, 11, 17, 23, 29, 30,

these being the number of zeros that cannot appear at the low order end of the factorials. For example:

23!	=	25852016738884976640000
24!	=	620448401733239439360000
25!	=	15511210043330985984000000

Whenever N contains a factor of 5, factorial N will have one more zero than did factorial (N-1). If N has a factor of 25, then two new zeros will be introduced into the table, as is seen above at N = 25. Thus, there is no factorial having 5 zeros at the low order end. The series involved thus goes:

5, 11, 17, 23, 29, 30, 36, 42, 48, 54, 60, 61, 67, 73, 79, 85, 91, 92, 98, 104, 110, 116, 122, 123, 129, 135, 141,...

Factorial 10000 has 2499 zeros at its low order end (see PC4-10). This figure checks as follows:

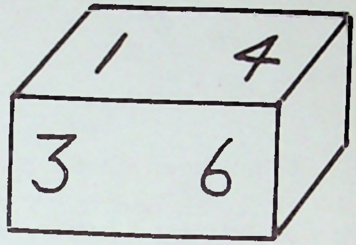
10000/ 5	=	2000
10000/ 25	=	400
10000/ 125	=	80
10000/ 625	=	16
10000/3125	=	3
		2499

Since $10000 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 16$, this implies that the series under discussion also contains the terms 2496, 2497, and 2498. □

Superominoes

PROBLEM 90

The accompanying chart shows the arrangements of numbers to be placed on the sides of 25 boxes, as shown here:



to form 4-sided "dominoes." All 100 combinations from 00 to 99 appear once each.

The boxes can be chained, as in dominoes, with adjacent numbers equal. For example, a chain can be made starting with box A1 side 4 and continuing with Y2-4, Q2-2, Y4-3, and K3-3, so that the numbers showing on the boxes are:

4-6-6-2-2-0-0-9-9-7-...

(the last two numbers illustrating that the boxes can be turned end-for-end.) It is relatively easy to form a chain of all 25 boxes. Is it possible to form such a chain so that the starting and ending digits are alike? That is, can all 25 boxes be chained in a circle?

	A	K	Q	T	Y
1	1 4 3 6 5 5 4 6	5 6 2 8 0 2 3 7	3 8 1 9 4 4 4 5	0 7 4 3 6 0 5 1	2 5 4 2 3 5 5 9
2	7 8 9 2 1 5 9 6	1 3 7 3 6 2 6 3	6 1 2 0 5 2 2 7	1 8 8 1 3 4 7 1	7 2 0 1 1 0 2 6
3	8 7 0 3 0 4 9 5	0 8 7 4 7 9 9 9	6 9 7 0 5 3 3 3	0 6 8 0 9 3 8 4	5 8 4 1 5 0 1 7
4	2 3 9 8 2 4 6 4	5 4 9 7 1 2 9 4	0 0 6 5 8 9 4 8	2 1 3 9 8 8 5 7	8 2 4 9 0 9 6 8
5	4 7 1 6 7 5 3 1	3 2 7 7 8 6 9 1	1 1 8 5 9 0 6 7	4 0 0 5 2 9 8 3	2 2 6 6 3 0 7 6

Zeros in Powers of 2

It has been shown that, for values of 2^T in decimal form, a value can be found having K adjacent zeros, for any K. For the first appearance of such numbers, the present state of knowledge is the following:

K	T
1	10
2	53
3	242
4	377
5	1491
6	1492
7	6801
8	14007
9	(greater than 60000)

For example, the 14007th power of 2 exhibits 8 adjacent zeros, starting at the 729th digit.

The accompanying table (from calculations made by Richard Lubin) shows that it is relatively easy to display strings of zeros of any length in powers of 2. The ones shown here are not, of course, the first appearances of strings of eleven zeros. □

T	2^T
10000000000000001	34687480163574218752
10000000000000002	69374960327148437504
10000000000000003	38749920654296875008
10000000000000004	77499841308593750016
10000000000000005	54999682617187500032
10000000000000006	09999365234375000064
10000000000000007	19998730468750000128
10000000000000008	39997460937500000256
10000000000000009	79994921875000000512
10000000000000010	5998984375000000102[illegible]
10000000000000011	1997968750000000204[illegible]
10000000000000012	39959375000000004096
10000000000000013	79918750000000008192
10000000000000014	59837500000000016384
10000000000000015	19675000000000032768
10000000000000016	39350000000000065536
10000000000000017	78700000000000131072
10000000000000018	57400000000000262144
1000000000000001	77407480163574218752
1000000000000002	54814960327148437504
1000000000000003	09629920654296875008
1000000000000004	19259841308593750016
1000000000000005	38519682617187500032
1000000000000006	77039365234375000064
1000000000000007	54078730468750000128
1000000000000008	08157460937500000256
1000000000000009	16314921875000000512
1000000000000010	32629843750000001024
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1000000000000016	88310000000000065536
1000000000000017	766200000000001310[illegible]
1000000000000018	53240000000000262144
1000000000000019	06480000000000524288

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